Theoretical Analysis of a Water Desalination System Using Low Grade Solar Heat

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Theoretical analysis of a solar desalination system utilizing an innovative new concept, which uses low-grade solar heat, is presented. The system utilizes natural means of gravity and atmospheric pressure to create a vacuum, under which liquid can be evaporated at much lower temperatures and with less energy than conventional techniques. The uniqueness of the system is in the way natural forces are used to create vacuum conditions and its incorporation in a single system design where evaporation and condensation take place at appropriate locations without any energy input other than low grade heat. The system consists of solar heating system, an evaporator, a condenser, and injection, withdrawal, and discharge pipes. The effect of various operating conditions, namely, withdrawal rate, depth of water body, temperature of the heat source, and condenser temperature were studied. Numerical simulations show that the proposed system may have distillation efficiencies as high as 90% or more. Vacuum equivalent to 3.7 kPa (abs) or less can be created depending on the ambient temperature at which condensation will take place. [DOI: 10.1115/1.1669450]

Keywords: *Low Grade Solar Heat, Solar Desalination, Vacuum, Solar Still, Desalination*

Introduction

Shortage of drinking water will be the biggest problem of the world in this century due to unsustainable consumption rates and population growth. Pollution of fresh water resources (rivers, lakes and underground water) by industrial wastes has heightened the problem.

The total amount of global water reserves is about 1.4 billion cubic kilometers. Oceans constitute about 97.5% of the total amount, and the remaining 2.5% fresh water is present in the atmosphere, polar ice, ground water and other inaccessible forms. This means that only about 0.014% is directly available to human beings and other organisms [1]. So, development of new clean water sources is imperative. Desalination of sea and/or brackish water is an important alternative, since the only inexhaustible source of water is the ocean.

Desalination processes require significant amounts of energy. Due to high cost of conventional energy sources, which are also environmentally harmful, renewable energy sources (particularly solar energy) have gained more attraction since their use in desalination plants will save conventional energy for other applications, and reduce environmental pollution.

Solar energy can be used to produce distillate directly in a solar still, which utilizes the greenhouse effect to evaporate the saline water. Design of a solar still requires optimization of many factors: brine depth, tight seal to prevent vapor leakage, thermal insulation, cover slope, shape and material of the still. A still requires frequent flushing of the salt water to prevent precipitation, which reduces its absorptivity and hence the efficiency. Still efficiency, defined as the energy used to evaporate the water to the solar energy incident on the still, is usually low and rarely exceeds 50%, with an average of 30–40% [2]. Simple solar stills have been subjected to numerous studies, aiming at improving the still efficiency. The effect of coupling the solar still to a flat plate solar collector was investigated by a number of researchers [3–10], which showed that the still performance could improve significantly, but of course the system cost will increase. Use of the latent heat of condensation to preheat the feed water has been studied [11,12], which improves the still efficiency. Evaporation at a low temperature, under vacuum conditions, leads to a good improvement in the system efficiency as was shown by many researchers [13–15]. Multi effect solar stills were investigated by [16,17], where the vapor from one stage condenses at the next stage, giving up its latent heat to that stage, thus evaporating part of the water from the second stage. Another way to increase the efficiency of the still is to minimize the heat losses to the environment [18,19]. Dutt et al. [20] investigated the effect of adding dye to the water, and showed that the output could be increased. The effect of adding a passive condenser to the still was studied [21–23], which showed an improvement in the unit performance.

This paper gives a theoretical analysis of an innovative new system, which uses low-grade solar heat. The system utilizes the barometric pressure to produce vacuum, which allows the design of a more efficient solar desalination system.

System Description and Operating Principle

The proposed desalination system consists of: a solar heating system, and an evaporation chamber and a condenser at a height of about 10 m above ground level, connected via pipes to a saline water supply tank and concentrated brine tank, and a fresh water tank, respectively. Figure 1 shows a schematic of the system. Vacuum is created by balancing the hydrostatic and the atmospheric pressures in the supply and discharge pipes.

Evaporation chamber has provisions to feed the cold fluid directly to the chamber and provide solar or other low-grade thermal energy through a closed loop heat exchanger as well as provisions to withdraw the concentrated brine. The incoming cold fluid and withdrawn brine pass through a tube-in-tube heat exchanger in order to extract the maximum possible energy from the hot brine. The evaporation chamber is connected to a condenser, which dissipates the heat of condensation to the environment.

It is known that the vapor pressure of seawater is about 1.84% less than that of fresh water over the temperature range of

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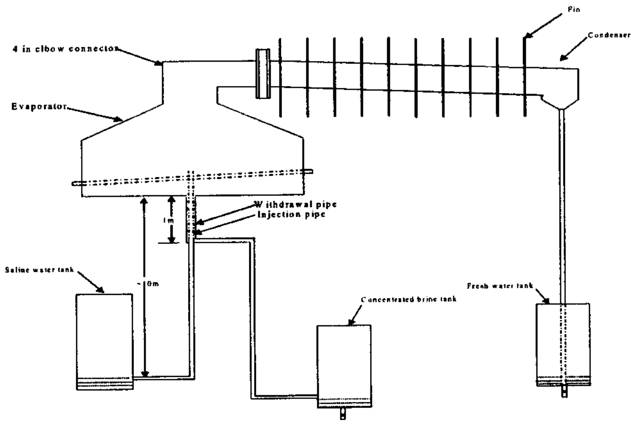

Fig. 1 Schematic of the system

0–100°C. This means that if the top of two chambers; saline water chamber (evaporator) and fresh water chamber (condenser) are connected while maintained at the same temperature, water will distill from the fresh water side to saline water side. In order to maintain the distillation of water from saline water side to fresh water side, the vapor pressure of the saline water must be kept above that of the fresh water. In the proposed process this will be done by increasing the temperature of the saline water utilizing solar energy. So, evaporation from saline water side to fresh water side is driven by the difference in the vapor pressures between the two sides.

To start up the unit, it will be filled completely with water initially, and the water will then be allowed to fall under the influence of gravity, in order to establish the vacuum. Depending on the barometric pressure, water will fall to a level of about 10 m from the ground level, leaving behind a vacuum.

Vacuum enables the desalination of water at a low temperature level, requiring a smaller amount of thermal energy. This heat will be provided from solar collectors, which will operate at a higher efficiency because of lower collector temperatures, which will minimize the heat loss to the environment. Simple flat plate collectors may be used to heat the saline water in the evaporator.

As saline water in the evaporator starts evaporating, its salinity increases which tends to decrease evaporation rate, so it becomes necessary to withdraw the concentrated brine at a certain flow rate and inject saline water at a rate equivalent to the withdrawal plus evaporation rates. The withdrawn water will be at a temperature equal to that of the evaporator, so it becomes necessary to recover the energy from it. A tube-in-tube heat exchanger will be used for this purpose, where injected water will flow inside the inner tube and withdrawn water will flow in the annulus in a counter current direction. The heat exchanger area will be such that a major part of the energy is recovered.

Under the influence of vacuum conditions at the saline water surface in the evaporator, water can be injected by the effect of atmospheric pressure; hence no pumping power is required. This makes the proposed system of a continuous process type, unlike flat basin solar still, which is usually a batch process. The withdrawn concentrated brine can be concentrated further and used to construct a solar pond, which may be used as a solar energy collection and storage system. The system will require periodic cleaning by flushing and restarting it. So, the non-condensable gases will not be allowed to accumulate to a degree of destroying the vacuum.

Theoretical Analysis

In this section a mathematical model that describes the processes in each component of the system is presented. A finite difference method was used to solve the set of coupled equations.

A complete mathematical description of the proposed distillation unit requires mass, energy and salt balances. In applying those balances, it is assumed that the heat capacity of the evaporator and the condenser materials is negligible and no temperature stratification occurs in the system.

Application of conservation of mass gives:

$$\rho_i \dot{V}_i = \rho_w \dot{V}_w + \rho_e \dot{V}_e \tag{1}$$

Application of solute conservation results in:

$$\frac{d}{dt}(\rho C V)_s = (\rho C)_i \dot{V}_i - (\rho C)_s \dot{V}_w \tag{2}$$

Application of the conservation of energy gives:

$$\frac{d}{dt}(\rho C_p V T)_s = Q_{in} + (\rho C_p T)_i \dot{V}_i - (\rho C_p T)_s \dot{V}_w - Q_e - Q_{loss} \tag{3}$$

The heat of evaporation will mainly be dissipated to the environment via the condenser during the process of vapor condensation and the rest will be carried away by the fresh water produced. So the condenser should be able to dissipate the amount of energy given by:

$$Q_e = \dot{m} h_{fg}^* \tag{4}$$

where, h_{fg}^* is the modified latent heat of condensation, given by [24],

$$h_{fg}^* = h_{fg} + 0.68 C_{pf} (T_s - T_{ci}) \tag{5}$$

This amount of heat is to be conducted through the condenser wall, and eventually transferred to the environment by convection (if we neglect radiative transfer). For heat conduction through the condenser wall,

$$\dot{m} h_{fg}^* = \frac{2\pi l_c k_c (T_{ci} - T_{co})}{\ln(r_{co}/r_{ci})} \tag{6}$$

For convective heat transfer to the ambient, the condenser is assumed to be a horizontal tube with circular fins. The rate of heat transferred from the condenser (fins and prime surface) may be calculated as,

$$Q_c = [h_{co,tip} N A_{f,tip} \eta_f + h_{co} N A_{f,sides} \eta_f + h_{co} A_b](T_{co} - T_a) \tag{7}$$

where the heat transfer coefficients were calculated from the relations given by Rohsenow et al. [25].

The various parameters required to solve the above equations are given in the appendix A.

The operating pressure of the proposed unit may be taken as the sum of the pressure in the vapor space at the point of condensation and the pressure drop occurring in the column. This pressure drop can be calculated by applying continuity equation and the energy equation between the inlet and outlet of the column:

$$(\rho A v)_{in} = (\rho A V)_{out} \tag{8}$$

$$\frac{P_{out}}{\gamma} + \frac{v_{out}^2}{2g} + z_{out} = \frac{P_{in}}{\gamma} + \frac{v_{in}^2}{2g} + z_{in} - h_L \tag{9}$$

In vacuum distillation there is larger volume of vapor to be handled. In order to avoid any significant impedance to vapor flow in the connecting pipes, the tubing should be as short and as wide as possible. It is often the practice to use connecting tubes of diameter larger than (1/10) of the evaporator diameter, at least in the upper part of the evaporator, where the lowest pressure prevails and flooding is most likely to take place [26]. This analysis assumes that evaporation is not impeded by foreign gas molecules.

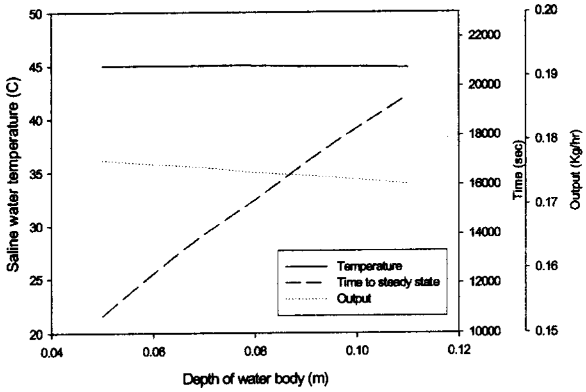

Fig. 2 Effect of depth of water body inside the evaporator on the unit performance

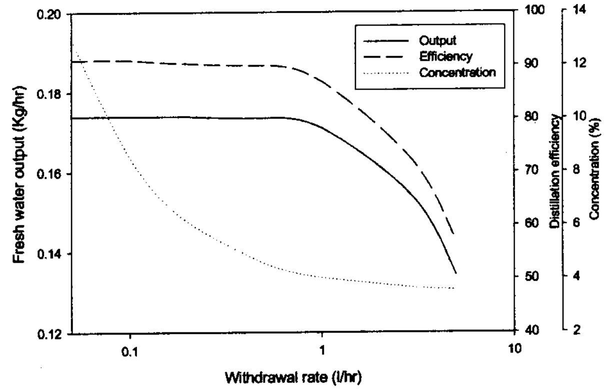

Fig. 3 Effect of withdrawal rate on the unit performance

Results

The mathematical relations presented in the theoretical analysis section were employed to determine the performance of the proposed unit. For all simulations, the following system specifications were assumed. The heat exchanger, through which the required thermal input is supplied to the saline water, is assumed to be a copper tube of 2.4 m length and 1.27 cm outside diameter. The evaporator is a cylinder of 0.2 m^2 cross sectional area, 0.2 m height, with a truncated cone fixed on top of it. The evaporator has a provision for feed water supply, through a 1.27 cm copper tube, partially enclosed by 2.54 cm PVC pipe which is used for withdrawing the concentrated brine. The two pipes form a tube-in-tube heat exchanger. There is also a provision to provide the additional required energy in the evaporator from a heat source through a heat exchanger. The condenser is a 4 inch copper tube of 0.5 m length, 0.25 cm thickness. On its lateral surface, 10 fins of 25.4 cm diameter and 0.0635 cm thickness are soldered 4 cm apart. The other end of the condenser is connected to a condensate receiver via 1.27 cm PVC pipe. A reference state was fixed as 25°C reference temperature, 3.5% reference solute concentration and 1021 Kg/m^3 reference density. For all calculations the ambient temperature was taken as 25°C. The heat transfer fluid through the evaporator heat exchanger was assumed to be water with a mass flow rate of 10 Kg/hr. This mass flow rate is equivalent to an optimum mass flow rate 50–75 Kg/hr.m^2 through solar collectors [27], assuming that in real life the unit will be supplied with its energy requirements from a solar collector with 1 m^2 collector area for each 1 m^2 evaporator area.

Operating conditions were varied to study the effect of those changes on the unit performance. The effect of depth of the water body was investigated with the withdrawal rate taken as 0.1 1/hr and the heat source temperature as 60°C. The results are shown in Fig. 2. This effect is limited to the time period required to get to the steady state. Once steady state is reached, the effect on the unit output and saline water temperature may be neglected. As the depth of water body varies from 0.05–0.11 m, the unit output varies only from 0.1769–0.1732 Kg/hr, which is almost constant. Therefore, a water depth of 0.1 m was used for the subsequent calculations. The effect of withdrawal rate on the evaporation rate is shown in Fig. 3. The distillation efficiency decreases from 91% for a withdrawal rate of 0.05 l/hr to about 57% at a withdrawal rate of 5 l/hr. For the same variation in the withdrawal rate, unit output decreases from 0.1739 to 0.1342 Kg/hr. The efficiency and output are almost constant in the withdrawal range of 0.05–1 1/hr, but looking at the concentration curve, which is based on steady state conditions in terms of salt concentration, we see that the concentration starts increasing rapidly if the withdrawal rate is less than 0.5 1/hr. At a withdrawal rate of 0.1 1/hr, the salt concentration at the steady state conditions will be about 8.58%, and if the withdrawal rate is reduced further, there will be a danger of scale formation. As the withdrawal rate goes above 1 l/hr, more energy will be carried away by the concentrated brine, hence more losses and less output will result. Therefore, a withdrawal rate of 0.1 l/hr was used, so as to keep the losses low and reduce the possibilities of scale formation.

The effect of fresh water temperature on the unit performance is shown in Fig. 4. Different values for the fresh water temperature were calculated by varying the condenser area, while keeping the heat source temperature constant at 60°C. The figure shows that as the temperature increases the unit output decreases, however, reducing that temperature below a certain value (slightly above the ambient) has a small effect in the unit output, but requires larger condenser. For example, to reduce the fresh water temperature from about 32°C where the unit output is about 0.1739 Kg/hr to 27°C where the unit output is about 0.1947 Kg/hr, the condenser area has to be doubled. The figure also shows that the pressure inside the unit increases as the temperature of the fresh water increases. A vacuum equivalent to 3.7 kPa (abs) or less can be created depending on the ambient temperature at which condensation will take place.

The effect of the heat source temperature was investigated over the range of 40–100°C. As expected, increasing the heat source temperature increases the saline water and fresh water temperatures, as can be seen from Fig. 5. For example, when the heat source temperature is 60°C, the steady state temperatures of the saline water and the fresh water will be about 44.9°C and 32.5°C respectively, and distillation efficiency will be about 91%. It is not only that the temperature in both chambers will be increased, but also the temperature difference between them. And as the temperature difference increases, the driving force for evaporation i.e. vapor pressure difference increases. Hence the unit efficiency will be improved.

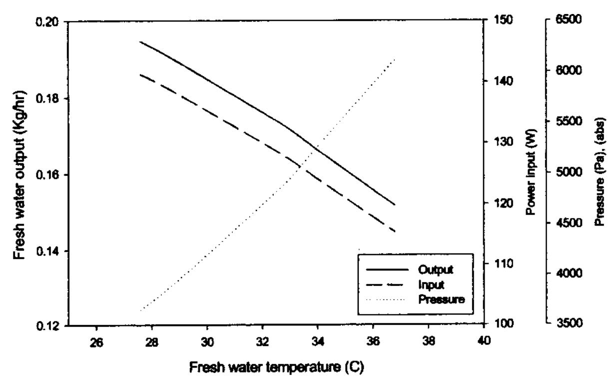

Fig. 4 Effect of fresh water temperature on the unit performance

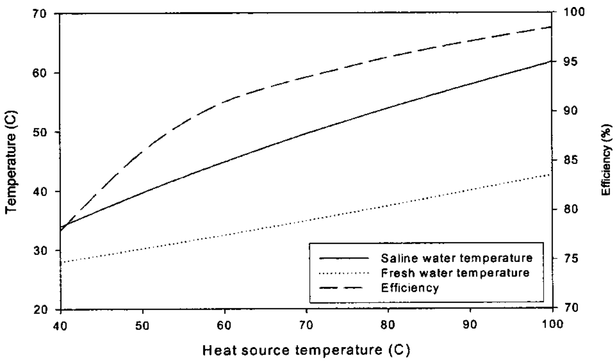

Fig. 5 Effect of the heat source temperature on the unit performance

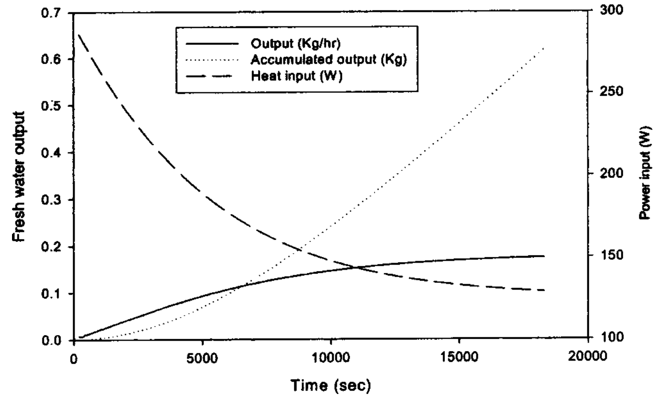

Fig. 8 Variation of input and output with time

Figure 6 shows that the heat input and the fresh water output of the unit, increase with the increase of the heat source temperature. The variation of saline and fresh water temperatures, operating pressure and solute concentration with time during the transient operation is shown in Fig. 7. Steady state conditions are assumed to be achieved if the saline water temperature does not vary more than 0.01°C during a time period of 100 seconds. With heat source temperature of 60°C, steady state conditions will be achieved after about five hours (if less rigorous convergence criteria is used, the time to reach steady state will be less), when the saline water temperature will be about 44.9°C, fresh water temperature about 32.5°C, and unit operating pressure about 4.8 kPa (abs). As shown in the figure, solute concentration will still be increasing by the time steady state conditions -based on saline water temperature- are achieved.

Variation of heat input and fresh water output with time during the transient operation is shown in Fig. 8. The heat input starts from a high value at the beginning, which is used mainly to raise the temperature of the saline water in the evaporator. The output increases until it reaches a steady state value of about 0.1739 Kg/hr. The accumulated output during the transient period is about 0.62 Kg.

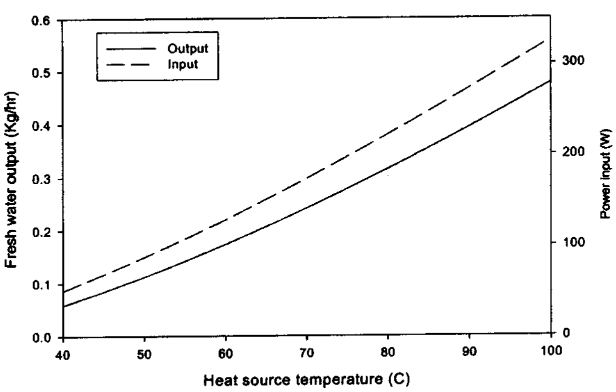

Fig. 6 Effect of the heat source temperature on the unit output

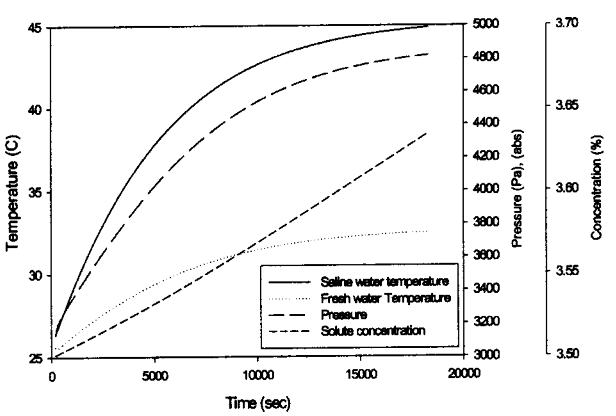

Fig. 7 Variation of saline and fresh water temperature, unit pressure, and solute concentration with time

Conclusion

An innovative water desalination system using low grade solar heat was studied. The system is of a continuous process type, unlike flat basin solar still, which is usually a batch process type. The results show that the proposed system may have distillation efficiencies as high as 90% or more, as compared to the conventional flat basin solar still, whose efficiency may reach 50% in the best operating conditions, with an average value of 30–40% [2].

Nomenclature

- A = Area (m^2)
- C = Solute concentration (%)
- C_p = Specific heat (J/kg.°C)
- g = Gravitational acceleration (m/s^2)
- h = Convection heat transfer coefficient ($W/m^2.K$)
- h_{fg} = Latent heat of vaporization (J/kg)
- k = Thermal conductivity (W/m.K)
- l = Length (m)
- $\dot{m}$ = Mass flow rate (kg/s)
- P = Pressure (Pa)
- Q = Heat transfer rate (W)
- T = Temperature (°C)
- U = overall heat transfer coefficient ($W/m^2.K$)
- V = Volume (m^3)
- $\dot{V}$ = Volumetric flow rate (m^3/s)
- v = Velocity (m/s)
- $\dot{\upsilon}$ = Evaporation rate per unit area (m/s)
- z = Elevation (m)

Greek Letters:

- ρ = Density (kg/m^3)
- γ = Specific Weight (N/m^3)
- η = Efficiency
- β_T = Volumetric thermal expansion coefficient ($=5*10^{-4}°C^{-1}$)
- β_C = Solutal expansion coefficient ($=8*10^{-3}\%^{-1}$)
- α_1 = Empirical coefficient (=0.0054, dimensionless)
- α_m = Empirical coefficient ($=10^{-7}-10^{-6}$ $kg/m^2.Pa.s.K^{0.5}$)

Subscripts:

a = Ambient
c = Condenser, collector
e = Evaporation
f = Fresh water or product
h = Hot; heat exchanger
i = Of the injection pipe; inside
in = Input
L = Loss
o = Outside, outlet
s = Of sea water chamber or the evaporator
w = Of the withdrawal pipe
O = Of the reference state

Appendix A

The evaporation rate per unit area from seawater chamber (evaporator) to fresh water chamber (condenser), may be written as [13],

$$\dot{\nu}_e=\frac{\alpha_m}{\rho_f}\left[f(C_s)\frac{P(T_s)}{(T_s+273)^{0.5}}-\frac{P(T_f)}{(T_f+273)^{0.5}}\right] \tag{10}$$

In the above equation the pressure drop between evaporator and condenser is neglected, however, this should be calculated and, if significant, added to that of fresh water, resulting in the following equation,

$$\dot{\nu}_e=\frac{\alpha_m}{\rho_f}\left[f(C_s)\frac{P(T_s)}{(T_s+273)^{0.5}}-\frac{P(T_f)+\Delta P}{(T_f+273)^{0.5}}\right] \tag{11}$$

Vapor pressure is given as:

$$P(T)=\exp(63.042-7139.6/(T+273)-6.2558\ln(T+273))*100 \tag{12}$$

The correction factor, which accounts for the solute concentration is given as:

$$f(C)=1-\alpha_1 C \tag{13}$$

Variation of density with temperature and concentration can be expressed by the following equation of state:

$$\rho(T,C)=\rho_0(1-\beta_T\Delta T_0+\beta_C\Delta C_0) \tag{14}$$

Solution concentration and temperature affect the value of the specific heat, which is given by [28]

$$\begin{aligned}C_p(T,C)=4186*\{&1.0049-0.0162C+3.5261*10^{-4}C^2\\&-[(3.2506-1.4795C+0.07765C^2)*10^{-4}T\\&+[(3.8013-1.2084C+0.0612C^2)*10^{-6}T^2\}\end{aligned} \tag{15}$$

Evaporation energy is given as:

$$Q_e=\rho_f h_{fg}(T_s)\dot{V}_e \tag{16}$$

The latent heat of vaporization of saline water is almost identical to that of fresh water [29], and is given by,

$$h_{fg}(T)=1000*[3146-2.36(T+273)] \tag{17}$$

Evaporation of water from saline water chamber tends to cool it, while condensation of vapor tends to heat fresh water, and for the process of distillation to be continuous, heat is to be added continuously to the evaporator and rejected from fresh water chamber. The amount of energy supplied to the system from the heat source, assumed to be in the form of hot water, via the heat exchanger can be calculated as:

$$Q_{in}=\dot{m}C_{pf}(T_{co}-T_w)\left[1-\exp\left(\frac{-\pi D_h U_h l_h}{\dot{m}C_{pf}}\right)\right] \tag{18}$$

The overall heat transfer coefficient, U_h , between the working fluid and saline water may be calculated as

$$U_h=\frac{1}{1/h_f+FT} \tag{19}$$

where FT is the heat exchanger fouling factor, which may vary in practice from 0.0005 for clean tubes to 0.001 W/m^2 .°C for adverse scale conditions [30], and h_f is the heat transfer coefficient between the collector fluid and the evaporator heat exchanger, which can be calculated as follows [24];

If the flow is laminar, i.e.

$$\mathrm{Re}_D=\frac{4\rho_0\dot{V}_h}{\pi D_h\mu}\leqslant 2300 \tag{20}$$

then,

$$Nu_D=3.66 \tag{21}$$

If the flow is turbulent, i.e.

$$\mathrm{Re}_D>2300 \tag{22}$$

then,

$$Nu_D=0.023\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.4} \tag{23}$$

The injection pipe, which carries the seawater to the evaporator, is coaxial and internal to the withdrawal pipe, so, major part of the energy of withdrawn water can be recovered. The temperature of the injected water, to the evaporator, can be calculated as follows [24]:

$$T_i=\frac{Q_h}{\rho_0\dot{V}_i C_{p0}}+T_0 \tag{24}$$

where $\dot{Q}_h$ is the actual heat transfer rate, given as

$$\dot{Q}_h=\varepsilon\dot{Q}_{\max} \tag{25}$$

$\dot{Q}_{\max}$ is the maximum possible heat transfer rate, given by

$$\dot{Q}_{\max}=C_{\min}(T_s-T_0) \tag{26}$$

ε is the heat exchanger effectiveness, given by

$$\varepsilon=\frac{1-\exp[-NTU(1-C_r)]}{1-C_r\exp[-NTU(1-C_r)]} \tag{27}$$

NTU is the number of transfer units, given as

$$NTU=\frac{UA}{C_{\min}} \tag{28}$$

$$C_r=\frac{C_{\min}}{C_{\max}} \tag{29}$$

C_{min} and C_{max} are the minimum and maximum of C_c and C_h, *respectively, where*

$$C_c=\rho_0\dot{V}_i C_{p0} \tag{30}$$

$$C_h=\rho_s\dot{V}_w C_{ps} \tag{31}$$

The product UA is given as

$$UA=\frac{1}{\dfrac{1}{2\pi r_i l h_i}+\dfrac{\ln(r_o/r_i)}{2\pi k l}+\dfrac{1}{2\pi r_o l h_o}} \tag{32}$$

The heat transfer coefficient between the injected water and the injection pipe, h_i , can be calculated from eq. 21 if the flow is laminar and from eq. 23 if the flow is turbulent.

The heat transfer coefficient between the withdrawn water and the withdrawal pipe, h_o , can be calculated as follows [24],

Table 1 Nusselt number for laminar flow in annulus [24]

D_i/D_o	Nu_i
0	-
0.05	17.46
0.1	11.56
0.25	7.37
0.5	5.74
1	4.86

If the flow is laminar, i.e.

$$\mathrm{Re}_D=\frac{\rho(D_o-D_i)}{\mu}\,\frac{\rho_s\dot{V}_w}{0.25\rho\pi(D_o^2-D_i^2)}\leqslant 2300 \tag{33}$$

then the value of Nusselt number can be selected from table 1.

If the flow is turbulent, eq. 23 can be used, with the diameter replaced by the hydraulic diameter [24].

The condensation heat transfer will mainly be dissipated to the environment via the condenser. So the condenser should be able to dissipate the amount of energy given by

$$\dot{Q}_c=\dot{m}(h_{fg}+0.68C_p(T_s-T_{if})) \tag{34}$$

where, h_{fg} is the latent heat of condensation and the second term in right hand side of the above equation accounts for the sensible heat transfer from the condensate.

This amount of heat is transferred through the condensate film, then conducted through the condenser wall, and eventually transferred to the environment by convection (if we neglect radiation).

Since the velocity of vapor is small, and the condensation rate is low, the condensate will flow as a thin annular film inside the tube. Then it flows in a longitudinal direction along the bottom side of the tube. For flow with Re<30,000, the average film heat transfer coefficient is given as [31]

$$\overline{m}_c=0.725\left[1-\frac{\theta}{\pi}\right]\left[\frac{h_{fg}^{*}gk_c^3(\rho_l-\rho_v)}{\gamma_l D(T_s-T_{if})}\right]^{1/4} \tag{35}$$

where, θ is the half angle made by joining the center of the tube to the two edges of the flowing liquid at the bottom.

Film properties involved are to be evaluated at an intermediate temperature between the interface, T_i, and the inside surface of the condenser, T_{ci}, so that the temperature jump across the film is accounted for [32]

$$T_{film}=T_{ci}+0.25(T_i-T_{ci}) \tag{36}$$

where T_i is the interface temperature calculated by assuming that all of the heat transferred from the vapor to the interface, $Q_{s\text{-}i}$, is conducted through the liquid film to the condenser surface, $Q_{i\text{-}ci}$. Therefore, the interface temperature does not change with time, i.e.

$$\frac{dT_i}{dt}=0 \tag{37}$$

and

$$\dot{Q}_{s-i}=\dot{Q}_{i-ci} \tag{38}$$

or

$$\dot{Q}_c=\overline{m}_c A_i(T_i-T_{ci}) \tag{39}$$

where A_i is the surface area of the liquid film at the interface for heat conduction through the condenser wall,

$$\dot{Q}_c=\frac{2\pi l_c k_c(T_{ci}-T_{co})}{\ln(r_{co}/r_{ci})}=\frac{T_i-T_{co}}{\dfrac{1}{2\pi r_{ci}l_c\overline{m}_c}+\dfrac{\ln(r_{co}/r_{ci})}{2\pi l_c k_c}} \tag{40}$$

where T_{co}, r_{ci}, r_{co}, l_c, and k_c are the condenser outside surface temperature, inside radius, outside radius, length, and thermal conductivity, respectively.

The condenser is assumed to be a horizontal tube with circular fins. The average heat transfer from the tips of the fins is given by [25]

$$Nu_s=cRa_s^b \tag{41}$$

This equation is valid for

$$2\leqslant Ra_s\leqslant 10^4 \tag{42}$$

and

$$1.36<1/\xi<3.73 \tag{43}$$

where

$$\xi=\frac{D_{co}}{D_{fin}} \tag{44}$$

Rayleigh number and other constants are given by

$$Ra_s=\frac{g\beta(T_{co}-T_a)S^3}{\alpha\gamma}\,\frac{S}{D_{fin}} \tag{45}$$

$$b=0.29$$

$$c=0.44+0.12/\xi,$$

and S is the distance between two successive fins.

Heat transfer from the cylinder and lateral fin surfaces is given by [25]

$$Nu_s=\frac{Ra_s}{12\pi}\left\{2-\exp\left[-\left(\frac{c1}{Ra_s}\right)^{3/4}\right]-\exp\left[-\beta\left(\frac{c1}{Ra_s}\right)^{3/4}\right]\right\} \tag{46}$$

where

$$1.67<1/\xi<\infty \tag{47}$$

$$\beta=0.17\xi+\exp(-4.8\xi) \tag{48}$$

$$c1=[23.7-1.1(1+152\xi^2)^{1/2}1+\beta]^{4/3} \tag{49}$$

The rate of heat transferred from the condenser (fins and prime surface) can be calculated as

$$Q_c=[h_{co,tip}NA_{f,tip}\eta_f+h_{co}NA_{f,sides}\eta_f+h_{co}A_b](T_{co}-T_a) \tag{50}$$

where N is the number of fins, and η_f is the fin efficiency calculated as follows [24],

$$\eta_f=C\frac{K_1(mr_{co})I_1(mr_{2c})-I_1(mr_{co})K_1(mr_{2c})}{I_0(mr_{co})K_1(mr_{2c})+K_0(mr_{co})I_1(mr_{2c})} \tag{51}$$

where

$$C=\frac{2r_{co}/m}{r_{2c}^2-r_{co}^2} \tag{52}$$

$$r_{2c}=r_{fin}+t/2 \tag{53}$$

$$m=\left(\frac{2h_{co}}{k_c t_{fin}}\right)^{1/2} \tag{54}$$

and I_0, I_1 and K_0, K_1 are the modified Bessel functions of the first and second kind, respectively.

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